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NUMERICAL SOLUTION OF SYSTEM OF SECOND ORDER DIFFERENTIAL EQUATIONS BY DIFFERENTIAL TRANSFORM METHOD

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ABSTRACT

In this paper, one dimensional Differential Transform Method (DTM) is applied on system of Second order differential equations. The approximate solutions obtained are in series form with easily computable components.

Three examples are presented and the proximity of the numerical results obtained, when compared with exact solutions derivable from application of Laplace transform method, demonstrates the efficiency and simplicity of the method.

KEYWORDS: Differential Transform Method, Laplace Transform Method, Simultaneous equations, second order differential equations

1. INTRODUCTION

The Differential transform method (DTM) is a semi analytical numerical method that uses Taylor series for the solution of differential equations.

It is an alternative procedure for obtaining the Taylor series solution of the given differential equation and is promising for various other types of differential equations.

By application of the method, it is possible to obtain highly accurate results or exact solutions for differential equations.

The concept of the differential transform method was first proposed by Zhou[7], who solved linear and nonlinear initial value problems in electric circuit analysis. In recent years, Abdel-Halim Hassan[1] used differential transform method to solve higher order initial value problems.

Ayaz[2] used DTM to find the series solution of system of differential equations.

Naharil and Avinash[5] applied DTM to system of linear differential equations. Hassan[1] compared series solution obtained by DTM with decomposition method for linear and nonlinear initial value problems and prove that DTM is a reliable tool to find the numerical solutions.

2. The Differential Transform Method:

The transformation of the k th derivative of a function with one variable is

$$U(k) = \frac{1}{k!}\left(\frac{d^k u(x)}{dx^k}\right) \text{ at } x = x_0 \qquad (1)$$

Where $u(x)$ is the original function and $U(k)$ is the transformed function and the differential inverse transformation $u(k)$ is defined by

$$u(x) = \sum_{k=0}^{\infty} u(k)(x - \chi_0^k) \quad (2)$$

When $x_0 = 0$, the function $u(x)$ defined in (2) is expressed as

$$u(x) = \sum_{k=0}^{\infty} u(k)x^k \quad (3)$$

Equation (3) shows the similarity between one dimensional differential transform and one dimensional Taylor series expansion.

The following fundamental theorems on differential transform method are handy:

Theorem 1:

If $u(t) = \alpha g(t) \pm \beta h(t)$, then $U(k) = \alpha G(k) \pm \beta H(k).$

Theorem 2:

If $u(t) = t^n$, then $U(k) = \delta(k-n)$ where $\delta(k-n) = \begin{cases} 1, & \text{if } k = n \\ 0 & k \neq n \end{cases}$

Theorem 3:

If $u(t) = e^t$, then $U(k) = \frac{1}{k!}$

Theorem 4:

If $u(t) = g(t)h(t)$, then $U(k) = \sum_{l=0}^{k} G(l)H(k-l)$

Theorem 5:

If $x(t) = x_1(t)x_2(t)$, then $X(k) = \sum_{k_1=0}^{k} X_1(k_1)X_2(k-k_1)$

Theorem 6:

If $x(t) = \frac{d^m x_1(t)}{dt^m}$, then $X(k) = \frac{(k+m)!}{k!} X_1(k+m)$

Theorem 7:

If $x(t) = e^{\lambda t}$, then $X(k) = \frac{\lambda^k}{k!}$, λ is constant.

Theorem 8:

If $x(t) = \sin(\alpha t + \beta)$, then $X(k) = \frac{\alpha^k}{k!}\sin(k\frac{\pi}{2} + \beta)$ where α and β are constants.

Theorem 9:

If $x(t) = \cos(\alpha t + \beta)$, then $X(k) = \frac{\alpha^k}{k!}\cos(k\frac{\pi}{2} + \beta)$ where α and β are constants.

3. NUMERICAL APPLICATIONS

Example 1:

Consider the following system of differential equations:

$$\frac{d^2x}{dt^2} - \frac{d^2y}{dt^2} + x - y = 5e^{2t} \quad (4)$$

$$2\frac{dx}{dt} - \frac{dy}{dt} + y = 0 \quad (5)$$

With the conditions $x = 1, y = 2, \frac{dx}{dt} = 0$, when $t = 0$.

Applying differential transform method to equations (4) and (5) using the above mentioned theorem, we obtain

$$(k+2)(k+1)X(k+2) - (k+2)(k+1)Y(k+2) + X(k) - Y(k) = 5\frac{2^k}{k!} \quad (6)$$

$$2(k+1)X(k+1) - (k+1)Y(k+1) + Y(k) = 0 \quad (7)$$

Using initial conditions $X(0) = 1, Y(0) = 2$ and $X(1) = 0$.

Put $k = 0$, we have $X(2) = -4, Y(1) = 2$.

$$k = 1,\ X(3) = \frac{1}{3}, Y(2) = -7.$$

$$k = 2,\ X(4) = -\frac{1}{6}, Y(3) = -\frac{5}{3.}$$

$$k = 3,\ Y(4) = -\frac{3}{4.}$$

The approximate solution when $n = 4$ (number of terms) using equation (3) is given by

$$x(t)=\sum_{k=0}^{4}X(k)t^{k}$$

$$y(t)=\sum_{k=0}^{4}Y(k)t^{k}$$

Thus $x(t)=1-4t^{2}+\frac{1}{3}t^{3}-\frac{1}{6}t^{4}.$

$$y(t)=2+2t-7t^{2}-\frac{5}{3}t^{3}-\frac{3}{4}t^{4}.$$

Using the Laplace transform method, the exact solution of example 1 are

$$x(t)=4\cos t-2\sin t-\frac{1}{3}\{8e^{-t}+e^{2t}\}$$

$$y(t)=6\cos t+2\sin t-\frac{4}{3}\{2e^{-t}+e^{2t}\}$$

Example 2.

Consider the following system of differential equations:

$$\frac{d^{2}x}{dt^{2}}+8x+2y=24\cos 4t \quad (8)$$

$$\frac{d^{2}y}{dt^{2}}+2x+5y=0 \quad (9)$$

With the conditions $x=y=0,\frac{dx}{dt}=1$ and $\frac{dy}{dt}=2,$ when $t=0.$

Applying differential transform method to equations (8) and (9) using the above mentioned theorem, we obtain

$$(k+2)(k+1)X(k+2)+8X(k)+2Y(k)=24\frac{4^{k}}{k!}\cos(\frac{k\pi}{2}) \quad (10)$$

$$(k+2)(k+1)Y(k+2)+2X(k)+5Y(k)=0. \quad (11)$$

Using initial conditions $X(0)=Y(0)=0, X(1)=1, Y(1)=2$

Put $k=0,$ we have $X(2)=12, Y(2)=0.$

$k=1,\ X(3)=-2, Y(3)=-2.$

$k=2,\ X(4)=-24, Y(4)=-2.$

$k = 3,\ X(5) = 1, Y(5) = \frac{7}{10}$

The approximate solution when $n = 5$ (number of terms) using equation (3) is given by

$$x(t) = \sum_{k=0}^{5} X(k) t^k$$

$$y(t) = \sum_{k=0}^{5} Y(k) t^k$$

Thus $x(t) = t + 12t^2 - 2t^3 - 24t^4 + t^5$.

$$y(t) = 2t - 2t^3 - 2t^4 + \frac{7}{10} t^5.$$

Using the Laplace transform method, the exact solution of example 2 are

$$x(t) = \frac{9}{10} \sin 2t + \frac{412}{105} \sin 3t - \frac{22}{7} \sin 4t.$$

$$y(t) = \frac{1}{5}\{3\sin 2t - 4\cos 2t + \frac{4}{3}\sin 3t + \frac{48}{7}\cos 3t\} - \frac{4}{7}\cos 4t.$$

Example 3:

Consider the following system of differential equations

$$5\frac{d^2x}{dt^2} + 12\frac{d^2y}{dt^2} + 6x = 0 \quad (12)$$

$$5\frac{d^2x}{dt^2} + 16\frac{d^2y}{dt^2} + 6y = 0 \quad (13)$$

With the conditions $x = \frac{7}{4}, y = 1, \frac{dx}{dt} = \frac{dy}{dt} = 0,$ when $t = 0.$

Applying differential transform method to equations (12) and $(13),$ using the above mentioned theorem, we obtain

$$5(k+2)(k+1)X(k+2) + 12(k+2)(k+1)Y(k+2) + 6X(k) = 0$$

$$5(k+2)(k+1)X(k+2) + 16(k+2)(k+1)Y(k+2) + 6Y(k) = 0$$

Using initial conditions $X(0) = \frac{7}{4}, Y(0) = 1, X(1) = Y(1) = 0.$

Put $k=0$, we have $X(2)=\frac{-12}{5}, Y(2)=\frac{9}{16}$.

$k=1,\ X(3)=Y(3)=0.$

$k=2,\ X(4)=\frac{903}{800}, Y(4)=-\frac{237}{640}.$

$k=3,\ X(5)=Y(5)=0.$

The approximate solution when $n=5$ (number of terms), using equation (3) is given by

$$x(t)=\sum_{k=0}^{5} X(k)t^{k}$$

$$y(t)=\sum_{k=0}^{5} Y(k)t^{k}$$

Thus $x(t)=\frac{7}{4}-\frac{12}{5}t^{2}+\frac{903}{800}t^{4}.$

$$y(t)=1+\frac{9}{16}t^{2}-\frac{237}{640}t^{4}.$$

Using the Laplace transform method, the exact solutions of example (3) are

$$x(t)=\cos(t\sqrt{\frac{3}{10}})+\frac{3}{4}\cos(t\sqrt{6}).$$

$$y(t)=\frac{5}{4}\cos(t\sqrt{\frac{3}{10}})-\frac{1}{4}\cos(t\sqrt{6}).$$

4. CONCLUSIONS

In this paper, we have applied differential transform method to system of second order differential equations. Three examples considered gave obtained results in Taylor series form. The smooth functional form of the solution over a time step is an added advantage of the method.

Thus the DTM is a simple and efficient method for obtaining numerical solutions of system of differential equations.

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